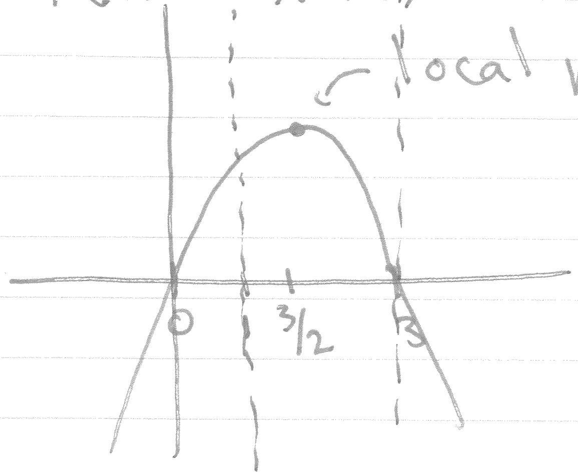
Global Extrema & Optimization

$f(x) = -x^2 + 3x = -x(x-3)$



$f'(x) = -2x + 3$

Find local extrema by setting $f'(x) = 0$.

$\longrightarrow x = 3/2$

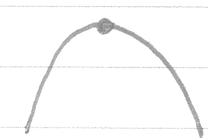
A Global maximum of $f(x)$ on an interval a —— b is a value x_{max} s.t.

$f(x_{max}) \geq f(x)$ for any

$a \leq x \leq b$

A local maximum of $f(x)$ is a point x_0 where

- $f'(x_0) = 0$



The global maximum is always

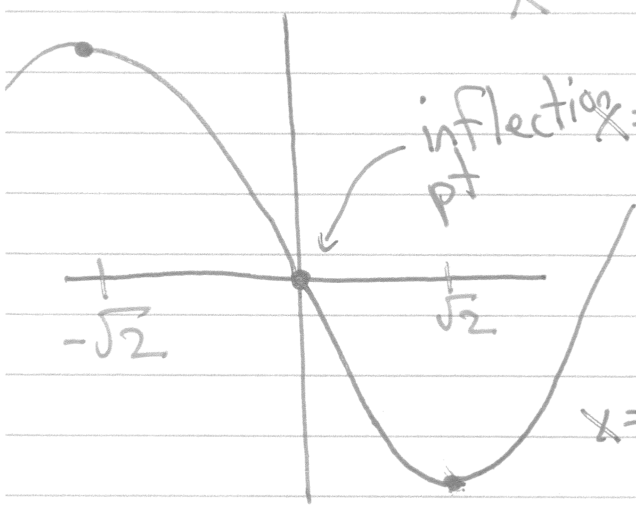
- A critical point, OR
- An endpoint.

$f(x) = x^3 - 6x, \ [-1, 7]$

• Find the critical points.

$f'(x) = 3x^2 - 6 = 0$

$x^2 = 2 \longrightarrow x = \pm\sqrt{2}$



$x = \sqrt{2} \longrightarrow f(x) = \sqrt{2}^3 - 6\sqrt{2}$

$= 2\sqrt{2} - 6\sqrt{2}$

$= -4\sqrt{2}$

$x = \sqrt{2} \longrightarrow f(x) =$

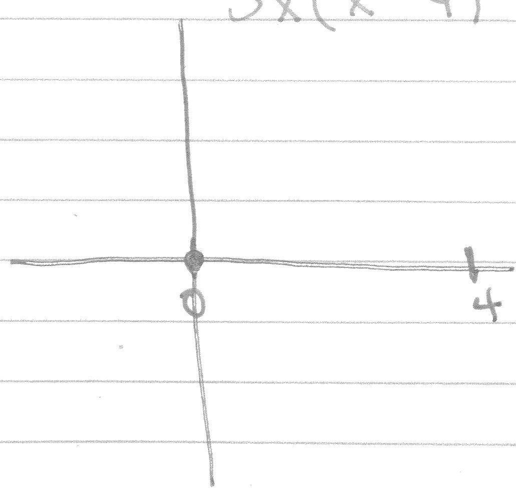
$\vdots$

$= 4\sqrt{2}$

$f(x) = x^3 - 6x^2, \ [-1, 7]$

$f'(x) = 3x^2 - 12x = 0$

$3x(x-4) = 0 \longrightarrow x = 0, \ x = 4$



$0 \rightsquigarrow 0^3 - 6 \cdot 0^2 = 0$

min

$4 \rightsquigarrow 64 - 6 \cdot 16 = -32$

$-1 \rightsquigarrow -1 - 6 \cdot 1 = -7$

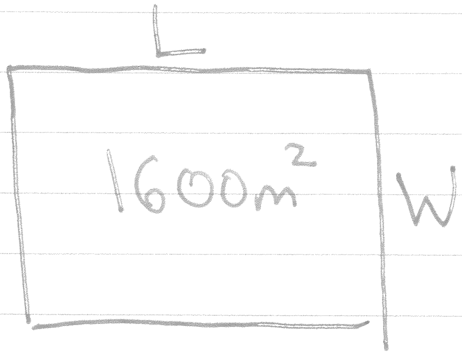
max

$7 \rightsquigarrow 343 - 6 \cdot 49 = \boxed{49}$

Constraint

Q: Rectangle of $\boxed{\text{area} = 1600\,m^2}$

Minimize the $\boxed{\text{perimeter}}$. function to minimize



Constraint: $L \cdot W = 1600$

Minimize: $2L + 2W$

- Use the constraint equation to write $2L+2W$ in terms of only one variable.

$L = \frac{1600}{W} \longrightarrow \boxed{\text{Minimize } 2\left(\frac{1600}{W}\right) + 2W}$

Global minimum over $(0, \infty)$

$\rightsquigarrow \cdots \quad f'(W) = -2 \cdot \frac{1600}{W^2} + 2 = 0$

$\rightsquigarrow W = 40$